The Effect of Correlated Clusters on Parameter Estimates in Multiple Membership Models

Yi Feng, Tessa L. Johnson, Laura M. Stapleton, & Yating Zheng (with special thanks to Tracy M. Sweet)

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UNIVERSITY OF MARYLAND

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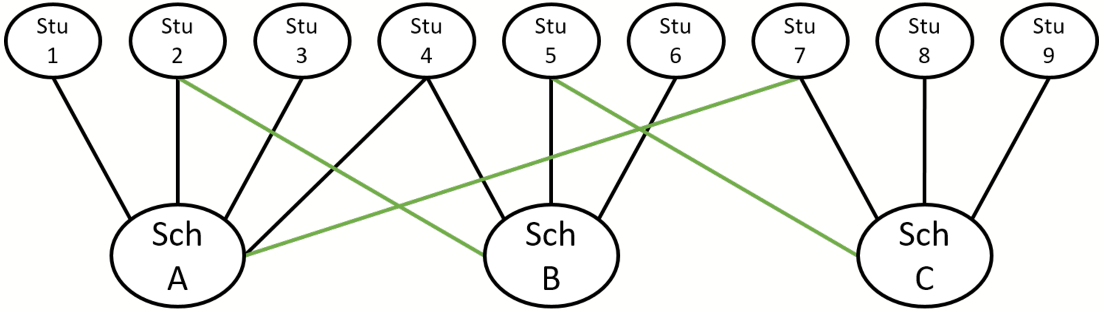
Overview

- Introduction to multiple membership
- The problem with non-random mobility patterns (with bonus path tracing activity)
- Simulation design & results
- The path forward

Real Data: Impure Nesting Structures

Longitudinal, multilevel education studies provide a wealth of information with implications for program evaluation and policy.

- These data are often quite complex in terms of their nesting structures (e.g., multiple membership)



Multiple Membership Model

$$\omega \sim N\left(Z_{\mathrm{W}} \cdot \beta,\ \tau_{00} \right)$$

$$y \sim N\left(\omega + X \cdot \gamma,\ \sigma^2 \right)$$

$\mathbf{Z_W}$ - weighted level-2 covariate matrix (weights sum to 1)

$\boldsymbol{\beta}$ - level-2 coefficient vector

$\boldsymbol{\tau}_{00}$ - variance of level-2 residuals

$\mathbf{X}$ - level-1 design matrix (covariates and constant)

$\boldsymbol{\gamma}$ - level-1 coefficient vector

$\boldsymbol{\sigma}^2$ - variance of level-1 residuals

Multiple Membership Model

Problem!

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- Weights are often assigned (not estimated) as 1/*H*, where *H* is the number of schools attended by student *i*
- A naive, first-school approach is a special case of this model where the first school is given a weight of 1 and subsequent school weights are set at 0
- $\mathbf{Z_W}$ is constructed as $w_{i,1}$* $z_{p,1}$ + … + $w_{i,H}$* $z_{p,H}$ - assumes 0 correlation between schools

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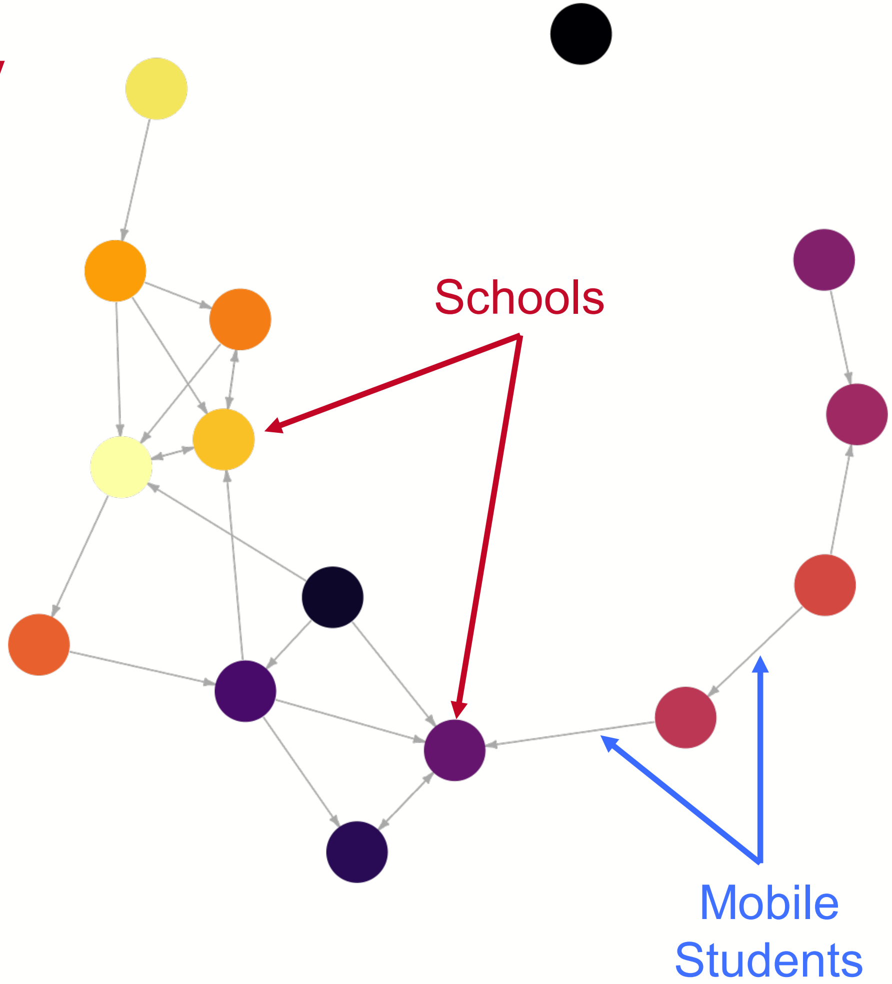
Patterns of Mobility

Students are mobile...but in a particular way

- Investigations of student mobility have found that clusters of schools form, passing students back and forth (Kerbow, 1996; Kerbow, Azcoitia, & Buell, 2003)



What do real data tell us? (SAT Math)

School residuals were calculated from a null model estimated on nonmobile students only. Correlations among residuals were then calculated between first and second, second and third, and first and third schools attended by mobile students.

Correlations Among School Residuals (n=266)	**1. n = 15926**	**2. n = 15185**	**3. n = 3902**
1. First School Attended	—		
2. Second School Attended	0.479	—	
3. Third School Attended	0.396	0.392	—

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Problem!

Let’s do some math…

Findings from empirical analyses reveal relatively large inter-school correlations, which impacts relevant modeling outcomes, such as ICC and level-2 variance.

Inter-School Correlation	Level-2 Variance	ICC	Composite ICC (across % mobility)		
			10%	25%	50%
Nonmobile	1.00	*0.314*	—	—	—
Mobile (0.0)	0.50	0.187	0.302	0.282	0.251
Mobile (0.2)	0.60	0.216	0.305	0.290	0.265
Mobile (0.5)	0.75	0.256	0.309	0.300	0.285

20% decrease

9% decrease

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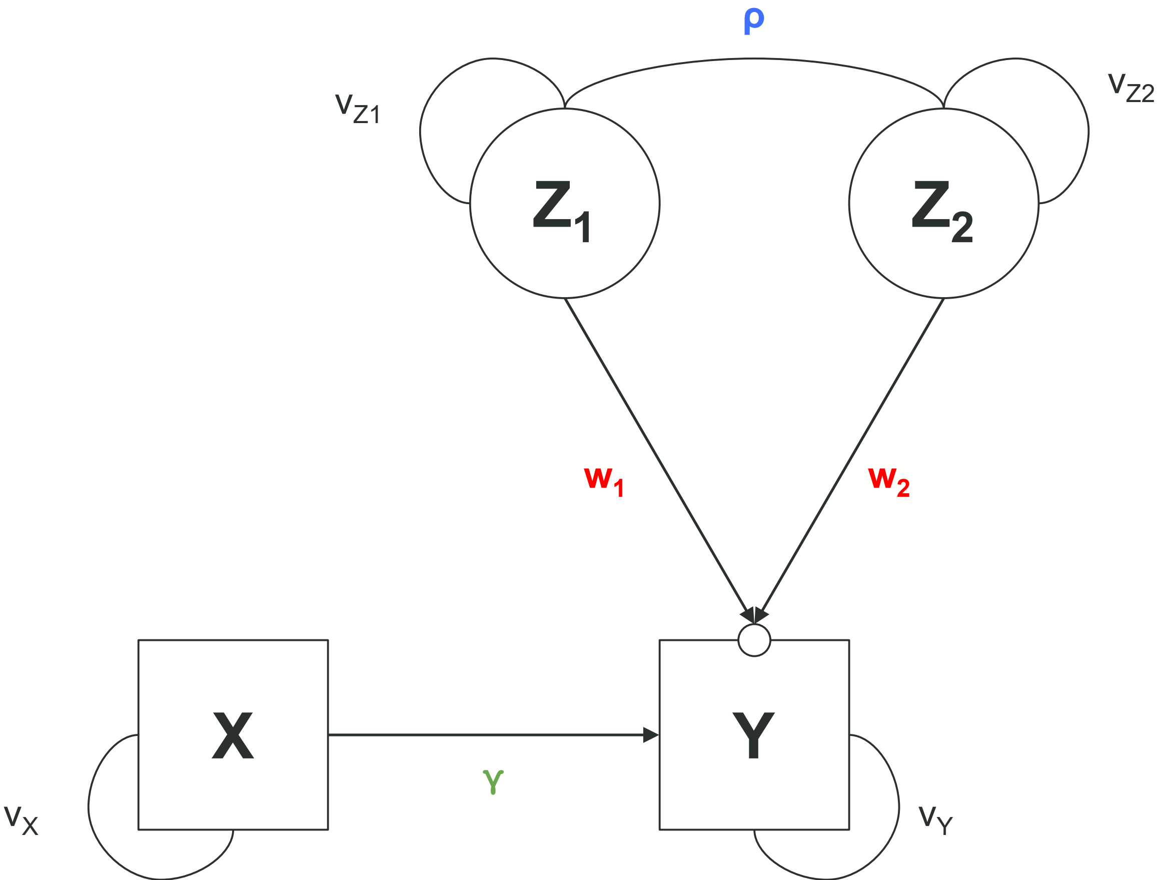
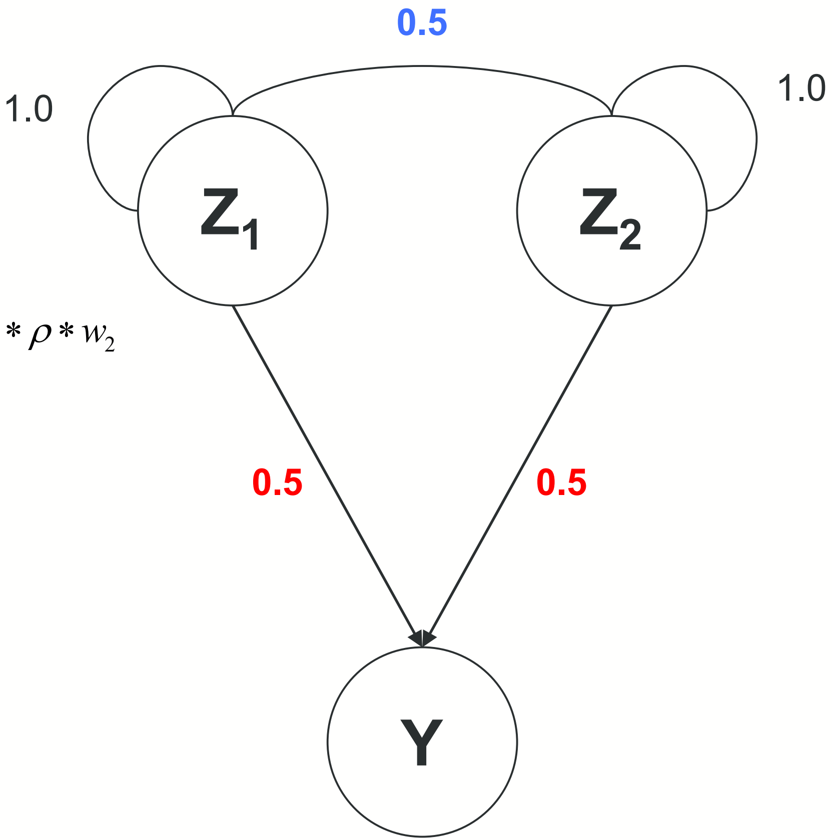
I’m sorry, what?

Let's do some path tracing!



Let's do some path tracing!

$$\mathrm{Var}(Y) = w_1 * v_{Z1} * w_1 + w_2 * v_{Z2} * w_2 + 2 * w_1 * \rho * w_2$$



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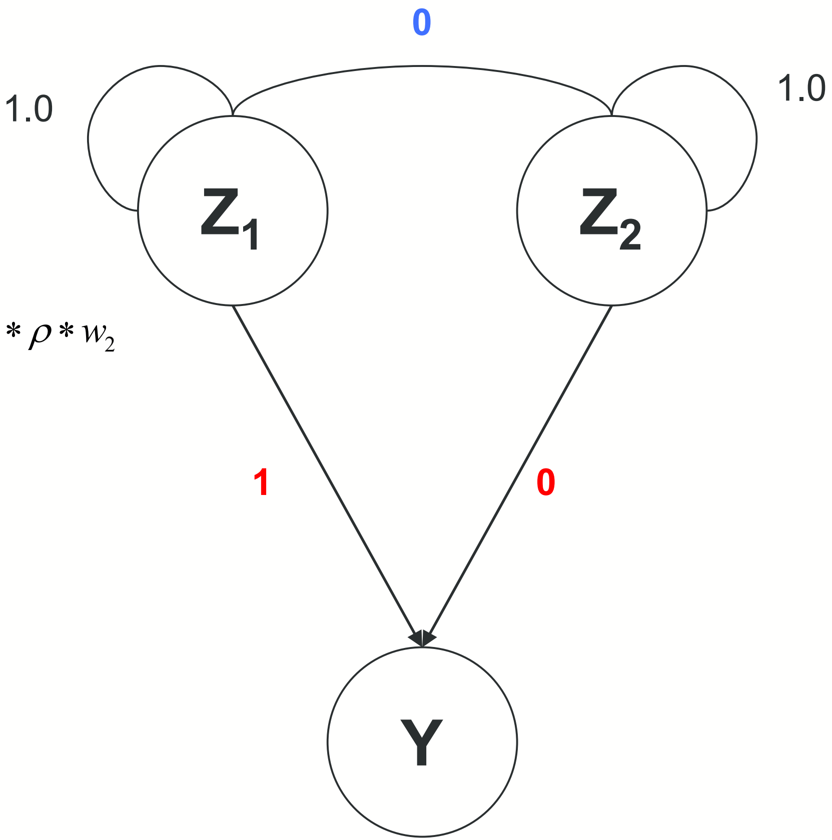

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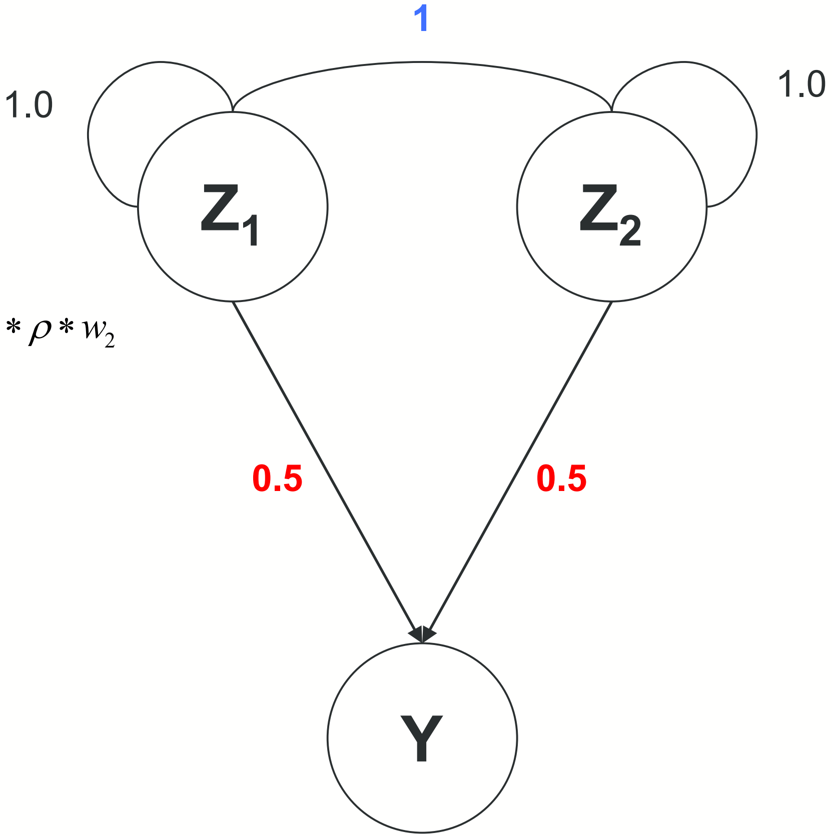
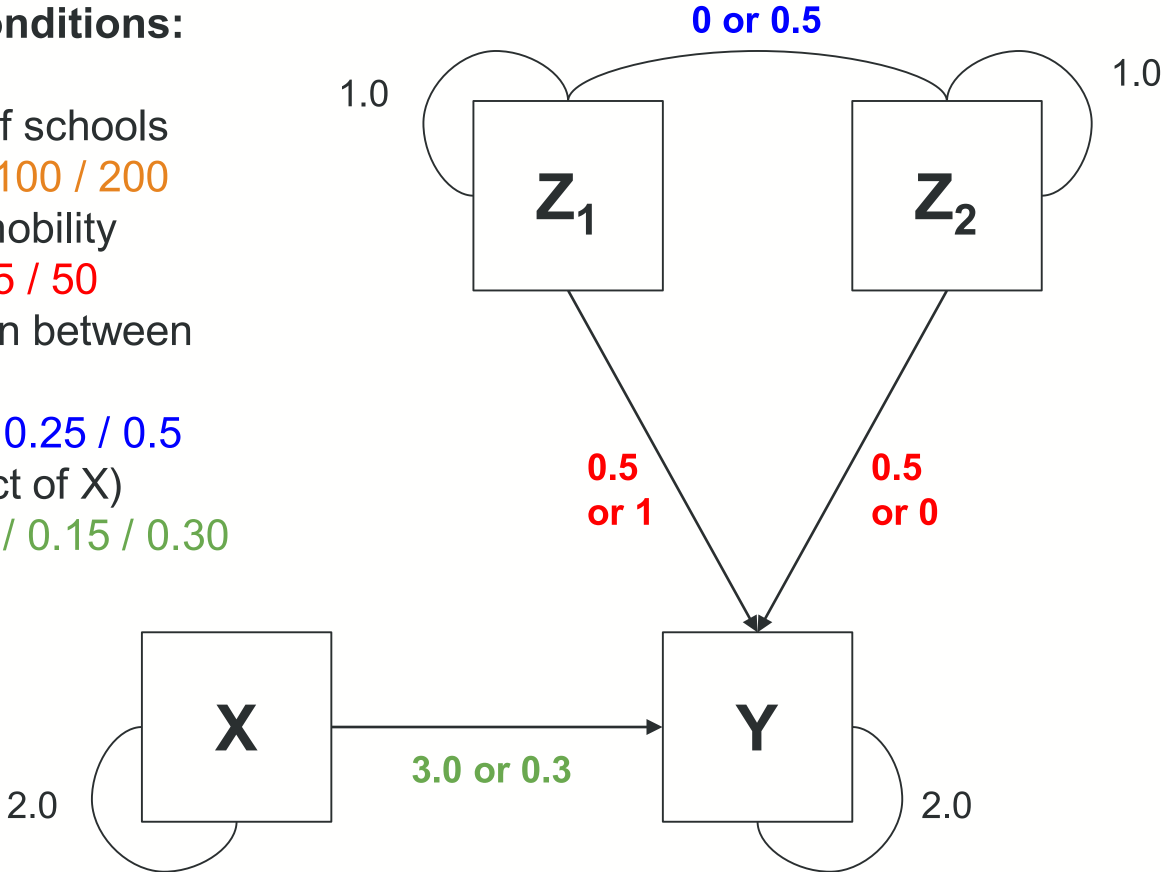


Simulation: Data-Generating Model

Simulation Conditions:

- Number of schools
  - 50 / 100 / 200
- Percent mobility
  - 0 / 25 / 50
- Correlation between schools
  - 0.0 / 0.25 / 0.5
- ICC (Effect of X)
  - 0.05 / 0.15 / 0.30

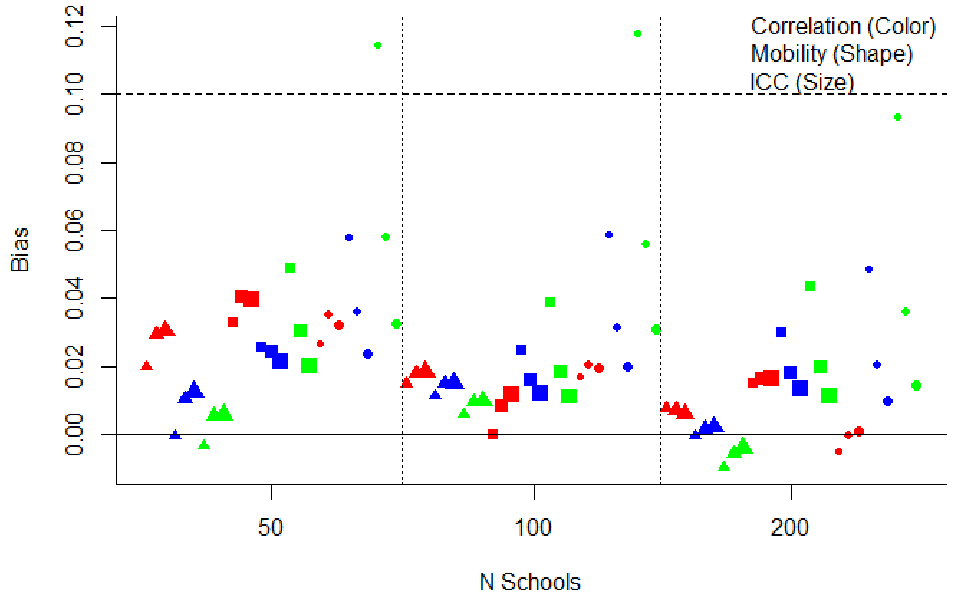


Relative Parameter Bias

Where do the models fail?

- High mobility (50%) &
- High correlation (0.50) &
- Low ICC (0.05)

Level-2 Variance Component



Not much of a problem!

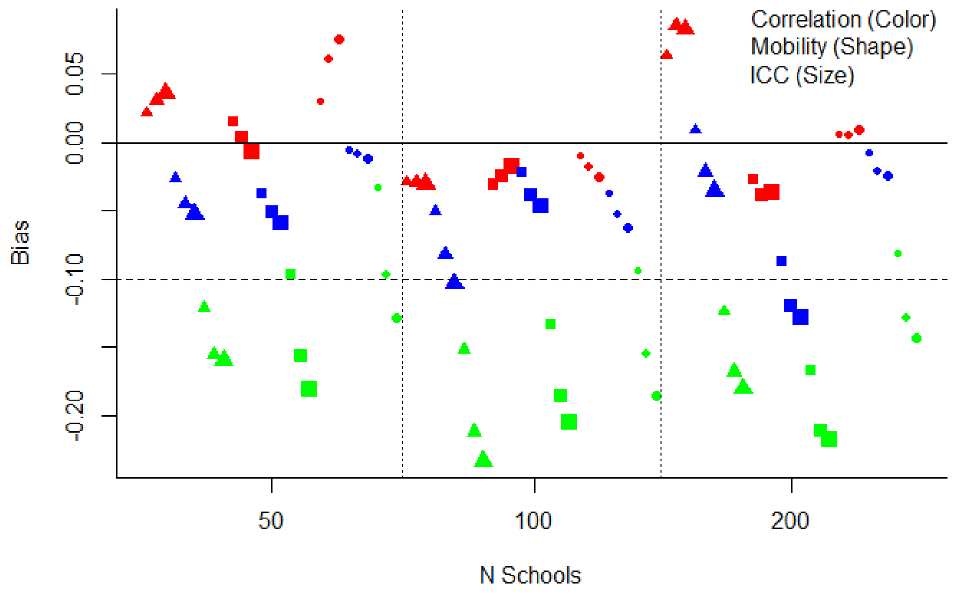
- So we’re good then, right?

Relative Std. Error Bias

Where do the models fail?

- High Correlation (all)
- Gets worse with increasing ICC

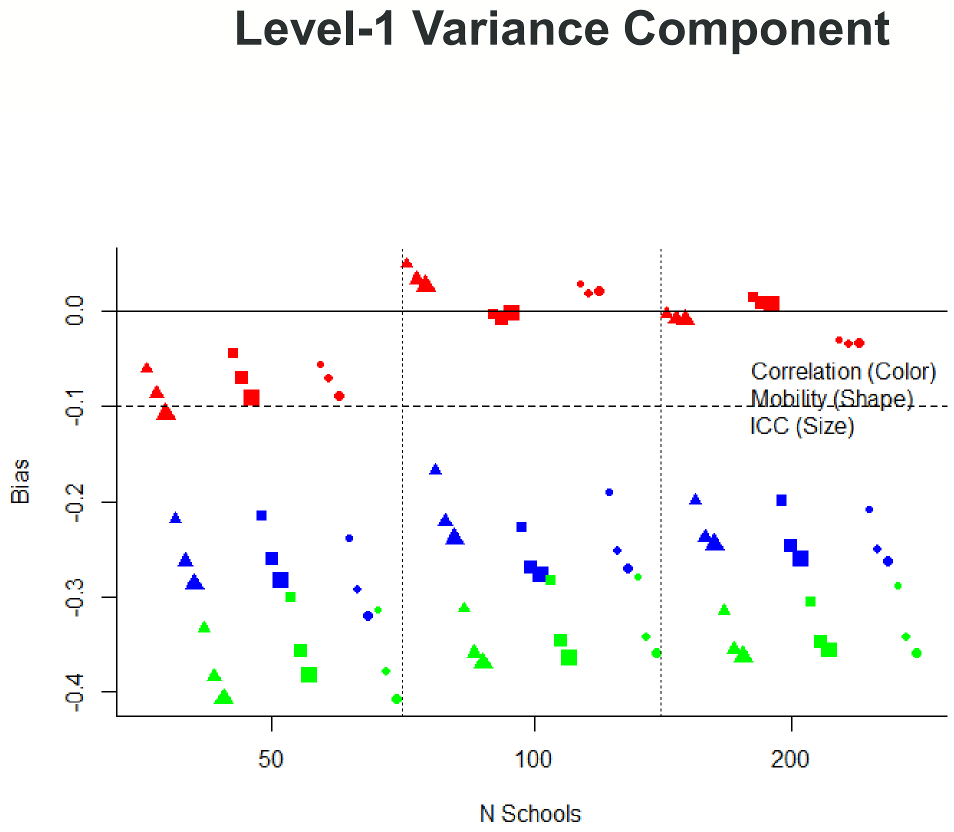
Level-2 Variance Component



Relative Std. Error Bias

Where do the models fail?

- High Correlations (0.25, 0.50)
- Gets worse with increasing ICC

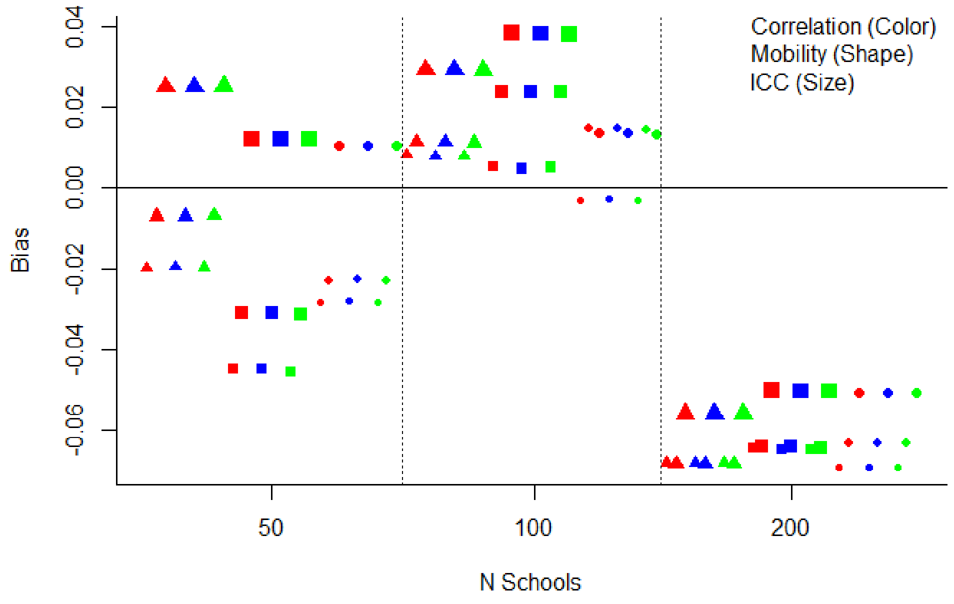


Relative Std. Error Bias

Where do the models fail?

- As may be expected, intercept fixed effects' standard errors are largely preserved

Intercept (Fixed Effect)



Results Summary

- Consistent with previous findings, fixed effects parameters and were not impacted by increasing inter-cluster correlations

- Level-2 variance estimates are biased upward when level-2 units are correlated (positive parameter bias)

- Standard errors of the variance components are severely underestimated when inter-cluster correlations are high

The Path Forward

- Even if mobility is not a variable of interest, it still has impacts on student outcomes
- Further, the correlations between mobile students’ schools will have a large impact on standard error estimation
- Future research will explore explicitly accounting for inter-school correlations in MMREM formulation or adjusting SEs
- Large-scale studies should make every effort to track students across schools; studies with large numbers of schools are not immune

References

- Chung, H., & Beretvas, S. N. (2012). The impact of ignoring multiple membership data structures in multilevel models. *British Journal of Mathematical and Statistical Psychology*, *65*(2), 185-200.
- Kerbow, D. (1996). Patterns of urban student mobility and local school reform. *Journal of Education for Students Placed at Risk*, *1*(2), 147-169.
- Kerbow, D., Azcoitia, C., & Buell, B. (2003). Student mobility and local school improvement in Chicago. *Journal of Negro education*, 158-164.
- Smith, L. J. W., & Beretvas, S. N. (2017). A comparison of techniques for handling and assessing the influence of mobility on student achievement. *The Journal of Experimental Education*, *85*(1), 3-23.

UNIVERSITY OF MARYLAND

Tessa L. Johnson
3942 Campus Drive, College Park, MD 20742
johnsont@umd.edu
@tessajolee